Written Exam for the M.Sc. in Economics summer 2012

Pricing Financial Assets

Final Exam

June 22, 2012

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The Exam consists of 3 problems that will enter the evaluation with equal weights.

Problem 1

Let the price of a traded stock, S, be modeled by the geometric Brownian motion

$$dS = \mu S dt + \sigma S dz$$

where μ and σ are constants, and where dt and dz are the standard short hand notations for a small time-step and a Brownian increment.

1. Describe the qualitative characteristics of this model, and discuss it's possible shortcomings.

2. Consider the transformation G of S given by the natural logarithm ($\ln$), i.e. $G(x) = \ln(x)$. Use Ito's lemma to find the process followed by $G(S)$.

3. Suppose that for $t = 0$ the stock price is S_0. What is the expectation of the natural log of the stock price at $t = T \geq 0$?

Problem 2

Suppose certain derivatives have values that depend on a single state variable given by the process

$$\frac{d\theta}{\theta} = m dt + s dz$$

where dz is a Wiener process.

1. Consider two such derivatives, assume a risk free interest rate of r, and use an arbitrage argument to derive and define the market price of ($\theta-$)risk, λ (You may assume that the prices of the derivatives follow geometric Brownian motions).

2. If θ is itself a traded asset, what can we say about the relation between m, s and the market price of risk?

Problem 3

1. In the Vasicek Model the (instantaneous) short term interest rate r is described by the process:

   $$dr = a(b - r)dt + \sigma dz$$

   where a, b and σ are constants, and dz a Wiener process. What does this mean for the behavior of the short term interest rate?

2. In this model the short term interest rate shows a predictable pattern. Why is this not necessarily incompatible with (informationally) efficient markets?

3. In the Hull-White Model of the short term interest rate r

$$dr = a(b(t) - r)dt + \sigma dz$$

where a and σ are constants, and dz a Wiener process ($b(t)$ is also written as $\frac{\theta(t)}{a}$). What is the purpose of the extra flexibility compared to the Vasicek Model?

4. Both the above models are one-factor models. What does that mean for the (instantaneous) correlation between changes in interest rates of different maturities (You may illustrate your point taking the Vasicek Model as an example)?